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**Financial Calculator Guide**

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Please feel free to distribute this document, and also please try to avoid printing out such a long document. The current version is [here](http://auapps.american.edu/~schrenk/FinCalcGuide.docx). Comments or corrections should be sent to Larry Schrenk.

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**Calculator Requirements**: You will need a calculator with financial functions for most finance classes. If you already have a TI-83 or 84 graphing calculator, that will do nicely. If not, you will need to buy a financial calculator. (Unfortunately, basic scientific calculators normally do not have financial functions. If in doubt, contact me.) *Do not spend a lot of money on a fancy model*. A basic financial calculator should cost $30-$40 at many department stores (maybe cheaper if you can get it online or used). A financial calculator has many financial functions (so you won’t need to memorize a lot of formulae for the exams). The most common models are the Texas Instruments BA II Plus and the Hewlett-Packard 10bII.

*BEWARE*: Do not buy the no-name financial calculator available at some stores for about $7.00–It does not work. Remember, if something appears to be too good to be true, it probably is.

*Important*: I use the Texas Instruments BA II Plus in these instructions, because it seems to be the most popular model. If you have the Hewlett-Packard 10bII or a TI-83/84 graphing calculator, please see the [appendix](#Appendix) *before* trying any problems with your calculator.

**Chapter 1–Single Dollar Problems: Present and Future Value**

There are three different methods for doing calculations: 1) using a financial calculator, 2) using formulae and a regular calculator and 3) using financial value tables and a regular calculator. We will only use the first (and easiest) method in this class, i.e., a financial calculator, so you may ignore the sections of the textbook that show how to use either formulae or tables. This gives you very little to read in the textbook, which is good because success with a financial calculator depends a lot of practice. This guide will have a few example problems and more will be assigned in individual topics. But first a small bit of theory…

**The Time Value of Money:** Everyone prefers getting $100 today to getting $100 in five years. But before we do any calculations, we should consider why this is so. We can isolate three separate motives for the time value of money, that is, three distinct reasons why you would prefer $100 today to $100 in five years:

1. *Inflation*: Prices go up, so if I wait five years I can buy less with the $100.
2. *Opportunity Cost*: By delaying the payment, I loose the opportunity to spend the $100 for the next five years.
3. *Risk:* In five years, you may not have the money to pay me, so waiting involves a risk of not getting $100.

The **interest rate** is how fast money grows (annually) over time. We get a **return** (or interest rate) on an investment to compensate us for waiting and accepting these three costs.

*Terminology*: In different contexts, the rate money grows over time may be called the ‘interest rate’, the ‘rate of return on an investment’ (often shortened to ‘rate of return’ or just ‘return’), the ‘compounding rate’ or the ‘discounting rate’.

**Time Lines**: When you are dealing with payments over time, the easiest way to visualize them is to draw a time line such as the one below.

**0 1 2 3 4 … N**

**Today**

**Time**

Today is 0. A year from now is 1, two years from now is 2, etc.

 Until you are comfortable with these problems, always start by drawing a timeline.

**Future Value Problems**

How much will you have in your bank account 5 years from now, if you deposit $100 today, earn 10% interest per year and make no additional deposits. (For clarity examples will be underlined.) In this time line X represents the amount in my account in five years:

**0 1 2 3 4 5**

**Today**

**Time**

***X***

**100**

This is a **future value** problem because I am asking the future value of a single dollar amount today. The amount deposited today is called the **present value**, and the process of going from the present value to the future value is called **compounding**. Future value problems often have the general pattern:

How much **future value (FV)** will you have after **N years**, if you deposit the **present value (PV)** today and get a **I/Y interest rate** per year?

We will use a financial calculator to solve these problems, but you should see the step by step calculations to understand what is going on.

*Year 1*: If start with $100.00, how much will I have after one year?

 $100.00 × 1.10 = $110.00

I multiply the $100 by 1 because I still have my $100 deposit and by 10% because that is the interest I receive for one year. We combine these in one calculation by multiplying the $100 by 1.10.

*Year 2*: I start the year with $110.00 in my account. How much will I have after one more year?

 $110.00 × 1.10 = $121.00

As in the first year, I multiply the starting amount by 1.10.

*Year 3*: I start the year with $121.00 in my account. How much will I have after another year?

 $121.00 × 1.10 = $133.10

You see that pattern: multiple each year by 1.10. I don’t need to do this in three different steps; instead, I can find the amount in year 3 from the original deposit:

 $100.00 × 1.10 × 1.10 × 1.10 = $100.00 × 1.103 = $133.10

This is what our financial calculator will do, but instead of worrying about formulae (yes, this one is easy and you could easily do it with a regular calculator, but the formulae get more complicated later), we just need to enter the data:

 N = 3 I/Y = 10 PV = $100.00

into a financial calculator.

**Future Value on your Calculator:** Now let’s use your calculator. Here are the most basic financial keys on the Texas Instruments BA II Plus–later we will add a few more:



How much do you have after 4 years if you deposit $200 today and the interest rate is 12%?

 N = 4 I/Y = 12 PV = 200

1. Press **4**, press **N**
2. Press **12**, press **I/Y**
3. Press **200**, press **+|-**, press **PV** (you get -200)
4. Press **CPT**, **FV** to get 314.70, i.e., $314.70

In 4 years, your bank account will have $314.70.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **4** | **12** | **-200** |  | **314.70** |

Notes:

* A number in red indicates a solution.
* You can enter the values for N, I/Y, and PV in any order.
* On some calculators, the interest rate key is labeled I, instead of I/Y.
* As in Excel, you enter 12% as 12 (don’t enter it as the decimal 0.12, because the calculator will think the interest rate is 0.12%).
* The present value must be entered as a negative value. (If you want to know why see [Why Enter Negative Values?](#Negative). Otherwise, just do it.)
* HP USERS ONLY: The Hewlett-Packard 10bII does not have a CPT key, just press FV and ignore all future references to a CPT key. If you still do not get the correct answer, you probably did not read the [HP appendix](#HP10BII).

**Future Value Practice Problems:**

1. How much is $350.00 worth in 5 years if the interest rate is 9%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **5** | **9** | **-350** |  | **538.52** |

1. How much is $400.00 worth in 15 years if the interest rate is 11%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **15** | **11** | **-400** |  | **1,913.84** |

1. How much is $1.00 worth in 100 years if the interest rate is 15%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **100** | **15** | **-100** |  | **1,174,313.45** |

**Present Value Problems**

I might also ask the reverse question: How much must you deposit today to have $100.00 in a bank account 5 years from now, if you earn 5% per year and make no additional deposits?

 In this timeline, X is the amount I need to deposit today:

**0 1 2 3 4 5**

**Today**

**Time**

**100**

**X**

This is a **present value** problem because I am asking the present (or current) value of a single dollar amount at some time in the future. The process of going from the future value back to the present value is called **discounting**. A present value problem typically has one of the two general patterns:

How much **present value (PV)** must you deposit today to have the **future value (FV)** after **N years** if you get an **I/Y interest rate** per year?

What is the **present value (PV)** today of the **future value (FV)** in **N years** if you get an **I/Y interest rate** per year?

These are actually identical questions–only the phrasing is different. In both you must find the present value of money coming in the future (given N and I/Y).

**Present Value on your Calculator:** How much is $200 received in 4 years worth now, if the interest rate is 12%?

 N = 4 I/Y = 12 FV = 200

1. Press **4**, Press **N**
2. Press **12**, Press **I/Y**
3. Press **200**, press **+|-**, press **FV** (you get -200)
4. Press **CPT**, **PV** to get 127.10, i.e., $127.10

The value today is $127.10. Or, suing different phrasing, you need to put $127.10 in the bank to have $200.00 in four years.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **4** | **12** | **-200** |  | **127.10** |

Notes:

* The future value must be entered as a negative value ([Why Enter Negative Values?](#Negative)).
* HP Users Only:If you did not get the correct answer now, see the [HP appendix](#HP10BII).

**Present Value Practice Problems:**

1. How much is $350.00 received in 5 years worth if the interest rate is 9%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **5** | **9** | **227.48** |  | **-350** |

1. How much is $400.00 received in 15 years worth if the interest rate is 11%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **15** | **11** | **83.60** |  | **-400** |

1. How much is $1,000,000 received in 100 years worth if the interest rate is 15%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **100** | **15** | **0.85** |  | **$1,000,000** |

**Resetting your Calculator**

When you do a second problem, your calculator does not automatically clear the values from the first problem. Depending on the type of problem, this may or may not cause an error in the second problem. To reset the BA II Plus:



1. Press **2nd**
2. Press **+|-**
3. Press **ENTER**

“RST 0.00” will appear on the display.

NOTE: I always refer to keys by their primary label, even if we are using their secondary function, so in step 2 I refer to the ‘+|-‘ key, not the ‘RESET’ key, even though we are using it for the reset function.

For resetting other calculators, see the [HP appendix](#HP10BII) or [TI-83/84 appendix](#TI83).

 I recommend that you reset your calculator at the start of each problem.

Now go practice!

**Chapter 2–Single Dollar Problems: Time and Interest Rate**

In the last topic you were introduced to present and future value problems. These problems have four variables:

* Present Value (PV)
* Future Value (FV)
* Interest Rate (I/Y)
* Years (N)[[1]](#footnote-1)

If you are given any three of these variables, you can use your financial calculator to find the fourth (unknown) value. In Chapter 1, we did problems in which FV or PV was the unknown; in the topic we will do problems in which I/Y or N is the unknown. The good news is that the calculator works the same for these new problems: put in three of the numbers and it will give you the fourth. The two new types of problems are interest rate and time problems.

**Interest Rate Problems**

Interest rate problems ask what interest rate is needed to meet a financial objective and often have the general pattern:

What **I/Y interest rate** do you need, If you want the **present value (PV)** to grow into the **future value (FV)** in **N years**?

**Interest Rate Problems on your Calculator**:

You want to buy a $30,000 car in 4 years. If you have $20,000 today, what interest rate do you need? What is I/Y?

1. Press **4**, Press **N**
2. Press **20000**, Press **PV**
3. Press **30000**, Press **+|-**, press **FV** (-30,000)
4. Press **CPT**, **I/Y** to get 10.67, i.e., 10.67%

You would need to get a return, i.e., an interest rate, of 10.67%.

Note: Here is the final rule about negatives–*One and only one* dollar value must be entered as a negative.

* If you are entering only one dollar value (as in present or future value problems), make it negative.
* If you are entering two dollar values (as in interest rate or time problems), make one of them negative–it does not matter which one.
* In this problem, you will get the same answer if you make the PV negative and the FV positive.
* You will get a wrong answer or an error message if you make both negative.
* [Why Enter Negative Values?](#Negative)

**Interest Rate Practice Problem:**

You now have $30.00 in an account in which you put $25.00 6 years ago. What interest rate have you been receiving?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **6** | **3.09%** | **-25** |  | **30** |

NOTE: When your answer is an interest rate, please express it in integer (10%), not decimal form (0.10), and *always include two decimal places* even if they are both zero, e.g., 10.00%.

**Time Problems**

Time problems ask how long it will take to meet a financial objective and often have the following general pattern:

How many **N years** will it take the **present value (PV)** to grow into **the future value (FV)** if you get an **I/Y interest rate**?

**Time Problems on your Calculator**:

You want to buy a $30,000 car. If you have $20,000 today and can get an interest rate of 12%, how long must you wait, i.e., what is N?

1. Press **12**, press **I/Y**
2. Press **20000**, press **PV**
3. Press **30000**, press **+|-**, press **FV** (-30,000)
4. Press **CPT**, **N** to get 3.58, i.e., 3.58 years

In time problems you must change the fraction of a year (here 0.58) into the approximate number of months (don’t worry about days). Multiply it by 12 (since there are 12 months in a year):

 12 × 0.58 = 6.96 ≈ 7

You would need to wait 3 years 7 months.

**Time Practice Problem:**

Your account grew from $50,000 to $75,000 with an interest rate of 10%. How long did you have the account?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **4.25** | **10** | **-50000** |  | **75000** |

Convert the fractional part of the year (0.25) to months by multiplying by 12, so 12 × 0.25 years = 3. The final answer is 4 years 3 months.

**Why Enter Negative Values? (Optional)**

Why do you need to make some of the dollar values negative? Take the simplest problem: How much do we have after 1 year if you deposit $100 and the interest rate is 10%?

You would set up and solve the following problem:

 FV = $100 × 1.10

Your calculator, however, sets up the same problem in a different way. It moves the FV to the other side of the equal sign and solves:

 FV + $100 × 1.10 = 0

If you don’t enter the present value as negative, -$100, the answer will have the wrong sign, i.e., your would get FV = -$110. In interest rate and time problems, you will get the wrong number or an error message (not just a sign change), if you do not enter one and only one dollar value as negative.

 Remember the rule: *One and only one dollar value must be entered as a negative*.

 WARNING: This rule applies to all the financial calculator problems we will do in this course. There are more complicated financial problems in which more than one of the dollar values must be entered as a negative, so if you tale another course in finance or do more complicated problems our simple rule may not be valid.

**Chapter 3–Annuities: Future and Present Value**

An **annuity** is a finite series of constant **payments** occurring at regular intervals. For example, I promise to pay you $10 every year for the next 25 years. There are three crucial features of any annuity:

* It is a *finite* series: it does not go on forever. There is a final payment; in the example, the last payment will be made in year 25.
* The payments are *constant*: every payment is the same amount, e.g., $10.
* The payments come at *regular intervals*: yearly, monthly, weekly, etc. In our example, the payments come every year.

An annuity is the typical way we save money: if I want to go on vacation, I would save $100 per week for a year, and this is an annuity. Timing like this is far more plausible than the earlier savings behavior when we put all the money in the bank today and just wait for it to grow. To have $1,000,000 at your retirement in 50 years (at 3% interest), you would need to deposit $228,107.08 today. I don’t think that is a very practical way for you to fund your retirement–unless, of course, you just happen to have an extra $228,107.08 under your mattress. You are far more likely to save a certain amount every pay period for many, many years; that is, your savings behavior is likely an annuity.

**Future Value Annuity Problems**

Saving money at regular intervals is a **future value annuity problem**, because I want to know how much I will have if I save $100 per week for a year.

Notes: For the moment we will only consider annuities that are annual; later we will do problems in which we save weekly or monthly. Also, we shall assume that the first payment beings next year, so my example would be more explicit if I promised to pay you $10 every year for the next 25 years *beginning next year*.

The timeline for a future value annuity looks like this:

**0 1 2 3 4 5**

**Today**

**Time**

***X***

**100**

**100**

**100**

**100**

To do annuities, we are going to need a new key on our calculators, so we can enter the amount of the constant payment. Appropriately, this key is labeled PMT which stands for payment.

Future value annuity problems often have the following general pattern:

How much **future value (FV)** will you have, if you make **payments (PMT)** for **N years** and get **I/Y interest rate** per year?

 Observe that you do not see present value mentioned in this question.

**Future Value Annuity Problems on your Calculator**: The mechanics of doing these on a calculator is simple, since you are just entering a value for PMT, instead of one for PV.

How much do you have after 3 years if you save $200 per year beginning next year and the interest rate is 12%?

 N = 3 I/Y = 12 PMT = 200

1. Press **3**, press **N**
2. Press **12**, press **I/Y**
3. Press **200**, press **+|-**, press **PMT** (you get -200)
4. Press **CPT**, **FV** to get 674.88, i.e., $674.88

In 3 years, your bank account will have $674.88.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **12** |  | **-200** | **674.88** |

Notes:

* The negative rule still applies: there is one dollar input (PMT) and we made it negative.
* We do not enter anything for the present value, so the present value remains at its default value of 0.
* If you did not reset your calculator before you started this problem, you might not get the right solution. If PV were set to anything but zero (that is from any earlier problem), you would not get the correct answer.

**Future Value Annuity Practice Problems:**

1. How much will you have if you save $100.00 per year for 25 years at 8%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **25** | **8** |  | **-100** | **7,310.59** |

1. How much will you have if you save $1000.00 per year for 5 years at 7%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **5** | **7** |  | **-1000** | **5,750.74** |

1. How much will you have if you save $1.00 per year for 50 years at 10%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **50** | **10** |  | **-1** | **1,163.91** |

**Present Value Annuity Problems**

Present value annuity problems are easy to calculate, but hard to understand at a more abstract level. Instead of asking how much a regular series of payments will be worth in the future, we are now asking how much a regular series of payments will be worth *today*. Think of this as a borrowing problem: how much could I borrow if I promised to make a series of regular payments into the future. For example, how much would a bank lend me if I promised to pay $200 per year for 3 years and the interest rate was 12%?

 **Future value** annuity problems are **savings** problems.

**Present value** annuity problems are **borrowing** problems.

Present value annuity problems often have the following general pattern:

How much **present value (PV)** can you borrow, if you make **payments (PMT)** for **N years** and get **I/Y interest rate** per year?

**Present Value Annuity Problems on your Calculator**: You are now entering a value for PMT, instead of one for FV.

How much could you borrow if you promised to pay $200 per year for 3 years and the interest rate was 12%?

 N = 3 I/Y = 12 PMT = 200

1. Press **3**, press **N**
2. Press **12**, press **I/Y**
3. Press **200**, press **+|-**, press **PMT** (you get -200)
4. Press **CPT**, **PV** to get 480.37, i.e., $480.37

You will be able to borrow $480.37.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **12** | **480.37** | **-200** |  |

Notes:

* Steps 1, 2 and 3 are identical for present and future value problems–the only difference is whether you press PV or FV in step 4.
* The negative rule still applies: there is one dollar input (PMT) and we made it negative.
* We do not enter anything for the future value, so the future value remains at its default value of 0.
* If you did not reset your calculator before you started this problem, you might not get the right solution. If FV were set to anything but zero (that is from any earlier problem), you would not get the correct answer.

**Present Value Annuity Practice Problems:**

1. What is the present value of $100.00 per year for 25 years at 8%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **25** | **8** | **1,067.48** | **-100** |  |

1. What is the present value of $1000.00 per year for 5 years at 7%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **5** | **7** | **4,100.20** | **-1000** |  |

1. What is the present value of $1.00 per year for 50 years at 10%?



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **50** | **10** | **9.91** | **-1** |  |

 The main difficulty in doing any annuity problem is figuring out whether it is a future value or a present value problem. Practice determining this and the calculation will be simple.

**Chapter 4–Annuities: Interest Rate, Time and Payment**

Just as we did in Chapter 2 for single dollar problems, in the case of annuities, we can also ask (besides PV and FV):

*What is the interest rate (I/Y)?* (Interest Rate Annuity Problem)

*How long will it take (N)?* (Time Annuity Problem)

But with annuities we can also ask:

*How much are my payments (PMT)?* (Payment Annuity Problem)

Unfortunately, I can ask each of these three questions with respect to *both* future value annuity problems *and* present value annuity problems. This means that before you can start calculating interest rate, time, or payment annuity problems, you still need to figure out whether it is a future value or present value annuity problem.

 This is complicated, so I organize the steps into a [flow chart](#Flowchart) a bit later. For now, a complementary set of examples should help:

This is the time question applied to a *future value* annuity problem (a savings problem):

If I save **payment (PMT)** every year, how many **N years** will it take to save the **future value (PV)** if I get a **I/Y interest rate**?

While this is the time question applied to a *present value* annuity problem (a borrowing problem):

If I make a **payment (PMT)** every year on a loan, how many **N years** will it take me to pay back a loan of **present value (PV)** if I have an **I/Y interest rate**?

 Before anything else, determine whether an annuity is a future value or present value problem. You will get an opportunity to practice this skill doing the financial calculator problems assigned in various topics.

**Interest Rate Annuity Problems**

Interest rate annuity problems ask you to find the interest rate to achieve a financial objective: For example, what interest rate do you need to have $2,000 in 3 years if you save $600 per year?

*First, is this a future value or present value annuity problem?* Look for key phrases in the problem: ‘save per year’ implies a savings problem and ‘in 3 years’ points to the future, so both suggest it is a future value annuity problem.

*Second, what result is the problem seeking? Which variable is unknown?* It is asking for the interest rate.

 N = 3 PMT = 600 FV = 2000

1. Press **3**, press **N**
2. Press **600**, press **PMT**
3. Press **2000**, press **+|-**, press **FV** (you get -2,000)
4. Press **CPT**, **I/Y** to get 10.73, i.e., 10.73%

You need an interest rate of 10.73% to reach your goal.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **10.73** |  | **600** | **-2000** |

**Time Annuity Problems**

Time annuity problems ask you hoe long it will take to achieve a financial objective: For example, how long do you have to save $600 per year to get $2,000 if the interest rate is 5%?

*First, is this a future value or present value annuity problem?* Look for key phrases in the problem: ‘save per year’ implies a savings problem, so it is a future value annuity problem.

*Second, what result is the problem seeking? Which variable is unknown?* It is asking for the time.

 I/Y = 5 PMT = 600 FV = 2000

1. Press **5**, press **I/Y**
2. Press **600**, press **PMT**
3. Press **2000**, press **+|-**, press **FV** (you get -2,000)
4. Press **CPT**, **I/Y** to get 3.16, i.e., 3.16 years

In time problems you must change the fraction of a year (here 0.16) into the approximate number of months (don’t worry about days). Multiply it by 12 (since there are 12 months in a year):

 12 × 0.16 = 1.92 ≈ 2

You would need to save for 3 years 2 months.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3.16** | **5** |  | **600** | **-2000** |

**Payment Annuity Problems**

Payment annuity problems ask you to find the constant payment that achieves a financial objective. The payment cab be the amount you need to save or the amount to repay a loan, depending on whether it is a future value or present value annuity problem.

For example, how much do you have to save per year to have $2,000 in 3 years if the interest rate is 12%?

*First, is this a future value or present value annuity problem?* Look for key phrases in the problem: ‘save per year’ implies a savings problem and ‘in 3 years’ points to the future, so both suggest it is a future value annuity problem.

*Second, what result is the problem seeking? Which variable is unknown? ‘*how much do you have to save per year’ is asking for the payment (PMT) variable.

 N = 3 I/Y = 12 FV = 2000

1. Press **3**, press **N**
2. Press **12**, press **I/Y**
3. Press **2000**, press **+|-**, press **FV** (you get -2,000)
4. Press **CPT**, **PMT** to get 592.70, i.e., $592.70

You need to save $592.70 per year.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **12** |  | **592.70** | **-2000** |

**Time Value of Money Decision Flowchart**

This flowchart should help you organize your approach to solving time value of money problems. Start at ‘Problem’ and work your way down the chart until you know 1) the type of problem you have and 2) the unknown value you must find.

*Problem Types*: Single Dollar, Future Value Annuity, Present Value Annuity

*Variables*: FV, PV, I/Y, N or PMT

*Step 1*: Is it a *single dollar* problem or an *annuity* problem?

*Step 2*: If it is a *single dollar* problem, what is the *unknown variable*?

FV, PV, I/Y or N (Done)

 If it is an *annuity* problem, is it *present value* or *future value*?

*Step 3*: If it is a *future value annuity* problem, what is the *unknown variable*?

FV, I/Y, N or PMT (Done)

 If it is a *present value annuity* problem, what is the *unknown variable*?

PV, I/Y, N or PMT (Done)

 Once you have some experience with your financial calculator, your main task will be to figure out the type of question being asked. Calculating is the easy part.

**Chapter 5–Non-Annual Compounding and Discounting**

While saving annually for the down payment on a home is better than having to put the present value in the bank today, most like we will save for things monthly, biweekly or weekly depending on the frequency of our paycheck. So far every financial problem has used one year as the basic unit, so we now need to adjust our calculators so that we can do non-annual calculations. This is to change the value for **payments per year** (P/Y or P/Yr). On the Texas Instruments BA II Plus, the following keystrokes will set the payments per year to monthly, i.e., P/Y = 12:

1. Press **2nd**, press **I/Y**
2. Press **12**
3. Press **ENTER**
4. Press **2nd**, press **CPT**

Change step 2 to 1 for annual, 26 for biweekly, or 52 for weekly.

Before you try a problem, you need one other detail. The N key stands for ‘Number’. But it is the number of **periods** (or payments), not the number of years that must be entered. When you are doing annual problems, as we have, these are the same, but not in non-annual problems.

A monthly annuity for two years has 24 periods: 2 × 12 = 24; a weekly annuity for one year has 52 periods: 1 × 52 = 52; a bi-weekly annuity for 4 years has 104 periods: 4 × 26 = 104.

How much do you have after 3 years if you save $200 *per month* beginning next month and the interest rate is 12%?

1. Set **P/Y** to **12**
2. Press **36**, press **N** (Remember 3 years × 12 months = 36 periods)
3. Press **12**, press **I/Y**
4. Press **200**, press **+|-**, press **PMT** (you get -200)
5. Press **CPT**, **FV** to get $8,615.38

You will have $8,615.38.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **36** | **12** |  | **-200** | **8,615.38** |

Once you have changed the payments per year, it should remain set even if you turn off your calculator. If the calculator is fully reset, you will need to do it again.

**Non-Annual Practice Problems:**

1. How much will you have if you save $100.00 per month for 25 years at 8%?
2. Determine that this is a *future value annuity problem*, because it involves saving.
3. Determine that the unknown variable is *FV*.
4. Set P/R to 12 (since the payments are ‘per month’).
5. Calculate number of periods: 25 years × 12 months = 300 periods.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **300** | **8** |  | **-100** | **95,102.64** |

1. How much can you borrow if you pay $50.00 per week for 5 years at 7%?
2. Determine that this is a *present value annuity problem*, because it involves borrowing.
3. Determine that the unknown variable is *PV*.
4. Set P/R to 52 (since the payments are ‘per week’).
5. Calculate number of periods: 5 years × 52 weeks = 260 periods.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **260** | **7** | **10,962.57** | **-50** |  |

1. How much do you need to save per month to have $10,000 in 5 years at 10%?
2. Determine that this is a *future value annuity problem*, because it involves saving.
3. Determine that the unknown variable is *PMT*.
4. Set P/R to 12 (since the payments are ‘per month’).
5. Calculate number of periods: 5 years × 12 months = 60 periods.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **60** | **10** |  | **129.14** | **-10000** |

If you are having trouble with steps 1 and 2, use the [flowchart](#Flowchart) and practice.

 Two suggestions: Before for every new calculation…

* Always reset your calculator.
* Always explicitly set the payments per year (P/Y) for the new problem.

These precautions will ensure that lingering numbers from your last calculation do not cause an error in any new calculation.

HP Users: You should already know how to do this. If not, see the [HP appendix](#HP10BII).

**Chapter 6–Accounting for Inflation: Real versus Nominal Rates**

One of the few important topics that the textbook does not cover in any detail is inflation. Inflation calculations are a complicated, but the issue is so important that you must be able to do them. Supplementary readings about inflation are assigned in this topic, so here we will only consider the calculations.

You would like to buy a Lexus LS in ten years which costs about $64,000. Using your financial calculator, you determine that you need to save $350 monthly to meet this goal (N = 10 × 12 = 120, I/Y = 8, FV = -64000). After ten years, you happily withdraw the $64,000 from your portfolio, walk into the dealer’s showroom with a bounce in your step, then stop dumbstruck when you see the sign on the Lexus LS with a price of $94,736–you are $30,736 short! If inflation were 4%, in the ten years you were saving, the cost of the car would go up by about 50%.

 If inflation were… The new price would be… You would be short…

 2% $78,016 $14,016

 3% $86,011 $22,011

 4% $94,736 $30,736

 5% $104,249 $40,249

 6% $114,614 $50,614

**The Effects of Inflation**

Our first task is to determine how to find future prices for different levels of inflation. Fortunately, you already know how to do this–even if you do not know that you know! In Topic 3 you learned to calculate how the value of money changes over time. To apply this to inflation, let I/Y stand for the *inflation* rate, rather than *interest* rate.

How much will a Lexus LS cost in 10 years if it costs $64,000 today and the annual inflation rate is 4%?

 N = 10 I/Y = 4 PV = 64000

1. Set **P/Y** to **1**
2. Press **10**, press **N**
3. Press **4**, press **I/Y**
4. Press **64000**, press **+|-**, press **PV** (you get -64000)
5. Press **CPT**, **FV** to get 94,735.63

In 10 years, a Lexus LS will cost $94,735.63.



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **10** | **4** | **-64000** |  | **94,735.63** |

For practice, check the other Lexus prices in the list above.

You would certainly be disappointed if you could not get the Lexus, but then imagine what would happen if you forgot inflation in your retirement calculations. If you are starting to save for retirement (as you should) in your 20s, there is 50 years worth of inflation to consider.

**Inflation-Adjusted Saving**

If we want to reach our long-term financial goals, we need to find the future cost of an item before we can calculate how to save for the purchase.

How much do you need to save weekly, if you want to buy an airplane in 6 years? The current cost of the plane is $95,000, the return on your investments is 7% and inflation is 3%?

Sub-Question 1: What will a $95,000 plane cost in 6 years, if inflation is 3%?

 N = 6 I/Y = 3 PV = 95000

1. Set **P/Y** to **1**
2. Press **6**, press **N**
3. Press **3**, press **I/Y**
4. Press **95000**, press **+|-**, press **PV**
5. Press **CPT**, **FV** to get 113,434.97

In 6 years, the plane will cost $113,434.97.

Sub-Question 2: How much do you need to save weekly, if you want to but an airplane in 6 years? The *future* cost of the plane is *$113,434.97*, the rate of return on your investments is 7%.

 N = 6 × 52 = 312 I/Y = 7 FV = 113,434.97

1. Set **P/Y** to **52**
2. Press **312**, press **N**
3. Press **7**, press **I/Y**
4. Press **113434.97**, press **+|-**, press **PV**
5. Press **CPT**, **FV** to get 292.79

Each week, you need to save $292.79.

 Inflation is normally calculated annually, but we are saving weekly, so the two sub-questions have different P/Ys.

Optional: The financial functions on your calculator can be used for non- financial calculations. The single dollar FV calculation, for example, can be used to calculate any change over time. The calculations we did in Sub-Question 1 would also solve the following non-financial problem: There are 95,000 residents in my home town. If the population grows at 3% per year, what will be the population in 6 years? The town will have 113,435 residents.

**Alternate Method (Optional)**

There is another want to do this same calculation. It is more difficult to understand, but easier to calculate. You are not required to know this method and if an inflation-adjusted saving problem is on an exam, you may use either method.

This method adjusts the interest rate to take care of the inflation problem. The interest rates we have been using are called ‘nominal’ interest rates. An interest rate that has been adjusted for inflation is called a ‘real’ interest rate (don’t worry about the terminology).

 Real Rate ≈ Nominal Rate – Inflation

Instead of doing a separate calculation to find the price of an item in the future (as above), we can use the real interest rate in our saving problem. Note: This formula is an approximation–note the ‘≈’, so the answer will be slightly different from the one in the first method.

How much do you need to save weekly, if you want to buy an airplane in 6 years? The current cost of the plane is $95,000, the rate of return on your investments is 7% and inflation is 3%?

 Real Interest Rate = 7% – 3% = 4%

 N = 6 × 52 = 312 I/Y = 4 FV = 95,000

1. Set **P/Y** to **52**
2. Press **312**, press **N**
3. Press **4**, press **I/Y**
4. Press **95,000**, press **+|-**, press **PV**
5. Press **CPT**, **FV** to get 269.53

Each week, you need to save $269.53.

Note: The approximation of the real interest rate causes this answer to differ from the $292.79 in the first method. On exams, you must show your work, so I will be able to tell which method you use and which result you should have.

**Appendix–Other Calculators**

**Hewlett-Packard 10bII:** There are three simple, practical differences between your calculator and the Texas Instruments BA II Plus used in the examples:

*Difference 1: Changing the Payments per Year (P/YR)*

You need to make a change in one of the HP default values (P/YR). (The discussion of [non-annual problems](#Topic_12) will eventually explain why you need to make this change):

1. Press **1**
2. Press **Orange**
3. Press **PMT**

If you ever do not get the correct solution to a problem, do this again. In general, however, once you make this change it should be fixed even if you turn off your calculator. If the calculator is fully reset, you will need to do it again. If you want to check that this change is in place:

1. Press **Orange**
2. Press **C**

The screen should briefly flash “1 P\_Yr”. If the screen flashes “12 P\_Yr”, go back and do the first procedure again.

*Difference 2: You don’t need (or have) a CPT key*

The Texas Instruments BA II Plus has a key marked CPT (‘Compute’). The Hewlett-Packard 10bII does not have a CPT key. Ignore any instructions to press CPT.

*Difference 3: Clearing the HP*

The instructions for clearing, i.e., returning the variables back to their default values (normally 0), is different for the HP. To clear your calculator:

1. Press **Orange**
2. Press **C**

This is the same procedure you used in Difference 1, to check the default value.

**Texas Instruments 83/84:** A graphing calculator works differently due to the larger screen. Users of the other calculators can only see one variable at a time, while you can see all the variables that are being used in the calculation. To do time value of money calculations:

1. Press ‘**apps**’ key twice. (This takes you to the TI-83/84’s TVM Solver.)
2. Enter the known values.
3. Put the cursor on the unknown value, i.e., the desired result.
4. Press **alpha** then **SOLVE**.

For example, to solve the future value problem: How much do we have after 4 years if we deposit $200 and the interest rate is 12%?

 N = 4 I/Y = 12 PV = 200

1. Press ‘**apps**’ key twice. (This takes you to the TI-83/84’s TVM Solver.)
2. Enter the known values:
	1. **N** = **4**
	2. **I/Y** = **12**
	3. **PV** = **-200**
3. Put the cursor on **FV**.
4. Press **alpha** then **SOLVE**.
5. You should get the result 314.70, i.e., $314.70

When discussing the other calculators, I have stressed the need to clear values, so that a value remaining from an earlier problem does not cause an error in a later problem. This is not crucial with the TI-83/84, since you can see all the values. If you start a problem and find that FV is already set to 1,333,333.00, zero it out, so that FV = 0, before you begin entering any new values.

1. Actually, N means number of periods which, in these problems, is the same as number of years. [↑](#footnote-ref-1)